Philosophy 211
Assignment 9

I. Prove these sequents.

1. $\exists x(Px \mathbin{\&} \forall y(x \neq y \rightarrow Rxy)) \vdash \forall x({\sim}Px \rightarrow \exists y(y \neq x \mathbin{\&} Ryx))$
2. $\exists x \forall y(x = y \rightarrow Px), \forall x \forall y((Px \mathbin{\&} Py) \rightarrow x = y) \vdash \exists x(Px \mathbin{\&} {\sim}\exists y(Py \mathbin{\&} x \neq y))$
3. $\forall x Rxx, \exists x \exists y \exists z(Rxy \mathbin{\&} Ryz \mathbin{\&} {\sim}Rxz) \vdash \exists x \exists y \exists z(x \neq y \mathbin{\&} x \neq z \mathbin{\&} y \neq z)$
4. $\exists x \forall y\ x = y, {\sim}\forall x Px \vdash \forall x {\sim}Px$

II. Which of the following sentences are true in these interpretations?

I1	I2
U: {a, b, c}	U: {a, b, c, d}
F: {a}	F: {b, c}
G: {a, b}	G: {a, b, d}
R: {⟨a,b⟩, ⟨a,c⟩}	R: {⟨a,b⟩, ⟨b,c⟩, ⟨c,d⟩, ⟨d,a⟩}

1. $\forall x(Fx \rightarrow Gx)$
2. $\exists x(Fx \mathbin{\&} \exists y({\sim}Gy \mathbin{\&} Rxy))$
3. $\forall x \exists y Rxy$
4. $\exists x(Fx \mathbin{\&} Gx) \rightarrow \forall x(Fx \text{ v } Gx)$
5. $\forall x \forall y((Gx \mathbin{\&} Gy \mathbin{\&} x \neq y) \rightarrow (Rxy \text{ v } Ryx))$

III. Construct countermodels to show that each of the following sequents is invalid.

1. $\forall x(Px \text{ v } Qx) \vdash \forall x Px \text{ v } \forall x Qx$
2. $\exists x(Px \mathbin{\&} Qx), \exists x(Qx \mathbin{\&} Rx) \vdash \exists x(Px \mathbin{\&} Rx)$
3. $\exists x \forall y Rxy \vdash \exists x \forall y Ryx$
4. $\forall x(Px \text{ v } Qx), \exists x{\sim}Px \mathbin{\&} \exists y{\sim}Qy \vdash \forall x(Px \rightarrow {\sim}Qx)$
5. $\forall x(Px \rightarrow \exists y(Qy \mathbin{\&} Rxy)), \forall x(Qx \rightarrow \exists y(Py \mathbin{\&} Rxy)) \vdash \forall x \forall y(Rxy \rightarrow Ryx)$
6. $\exists x \exists y(Px \mathbin{\&} Py \mathbin{\&} x \neq y), \forall x(Px \rightarrow Qx) \vdash \forall x(Qx \rightarrow \exists y(Py \mathbin{\&} Ryx))$